Inventory Control

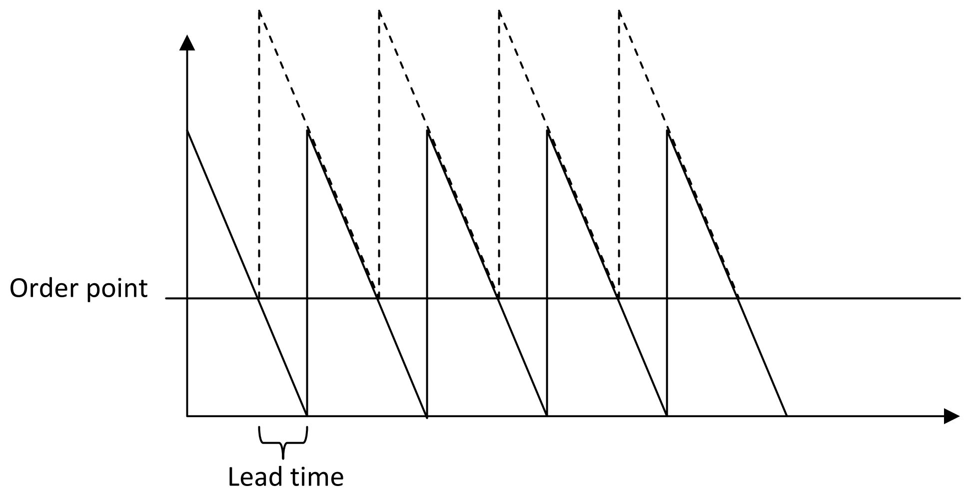
Background Information on VBA Programming in Business Economics by Sanne Wøhlk

Inventory Control is a classical area within business economics. Much research has been made in the area and many textbooks have been written on the subject. We refer the reader to Silver, Pyke, and Peterson, 1998 for a thorough presentation. Furthermore, the reader can consult almost any introductory book about management science for an introduction to the subject, for example Bozarth and Handfield, 2006. In this text we introduce the basic model and a few variations.

In the basic model, which is sometimes referred to as the EOQ model, we consider an inventory of infinite size, the price of each unit is p and the demand during a period of time is D units. In the basic model D is constant, but in many other models D is stochastic and is often assumed to follow a Normal Distribution. The demand is assumed to be continuous during each period. An *ordering cost* of size S is paid each time an order is placed. S is independent of the size of the order. An *inventory cost* c_h is paid per item per time period the item is in the inventory.

The *Order point* (or Re-order point) is the number of items in the inventory that causes an order to be placed. Hence every time the inventory level reaches the order point, an order of Q items should be placed. This is illustrated below.

The *lead time* is the time elapsed from an order is placed until the items arrive. The lead time is assumed to be constant. In the basic model, because demand is deterministic, the lead time can be assumed to be zero. In more advanced models where demand is uncertain, the lead time is important. In the basic model, backorder is not allowed, i.e. all demand must be fulfilled immediately.



In the basic model, the optimal value of Q, also called the economic order quantity, can be found as follows: First note that in a deterministic model, orders should arrive exactly when the inventory is empty. Hence with an order size of Q, the average inventory level is $Q/2$. Therefore, the average inventory cost per time period must be $c_h \frac{Q}{2}$. If D items are withdrawn from the inventory during each time period and Q items are ordered at a time then $\frac{D}{Q}$ orders must be placed during each time period. Hence, the ordering cost per period is $S\ \frac{D}{Q}$. Thereby the total cost per period is $c_h\ \frac{Q}{2} + \ S\ \frac{D}{Q}$. To find the value of Q that minimizes this expression, we have $\frac{c_h}{2} - \ S\ D\ Q^{-2}$ = 0, and thereby $Q_0 = \ \sqrt{\frac{2\ S\ D}{c_h}}$, which is the optimal order size.

In the basic EOQ model, the order point is set equal to the demand during the lead time, such that the order will arrive exactly when the inventory is empty. In models where the demand is stochastic, the EOQ order size is often used, but the order point is higher to reduce the risk of not being able to satisfy demand. As a general rule, the higher the order point, the higher the service level.

In stochastic models, unsatisfied demand can be handled in different ways: Unsatisfied demand can be lost with a cost incurred per unit lost demand or it can be backordered, i.e. satisfied as soon as there are items available. In the latter case, a *shortage cost* c_s per item per time period is used. Alternatively, unsatisfied demand can be satisfied by a secondary source at a higher price. Each way leads to a different model.

In the text, we also consider a more exotic inventory model where the procurement price of the items changes randomly. In this model, the order quantity should depend on the price as well as the inventory level. This model is considered from an analytical perspective in Larsen and Wøhlk.

Bibliography

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