Seat Reservation Problem

Background Information on VBA Programming in Business Economics by Sanne Wøhlk

The Seat Reservation Problem was first defined in Boyar and Larsen, 1999, where various strategies for solving the problem were compared using the so-called competitive ratio.

The problem considers a train that is traveling from a start station to an end station, stopping at a number of stations inbetween. The stations are numbered 1 through *k*. It is assumed that the train has *n* seats available. A series of requests for seats arrives in real time. Every request is for a single seat and states the start station, *s* and the end station, *t* for the person's journey. A decision must be made immediately after the arrival of the request to either accept the request or reject it. If the request is accepted, the reservation must be assigned to a seat and this decision cannot be changed subsequently. This decision must be made without any knowledge about future requests. The goal of the problem is often to accept as many requests as possible.

In the basic model, a rule is used stating that if a request can be accepted then it must be so. Hence if there is a seat available for the whole journey from *s* to *t*, then the request may not be rejected. This rule can be relaxed in variations of the problem. Doing that makes it possible to reject bad requests in the hope that better ones will arrive later. A variation of the problem has been studied where a limited number of planned seat changes are allowed. We refer the reader to Boyar, Krarup, and Nielsen, 2004 for details on this problem.

Even though the problem is defined for reservation of seats in a train, many other reservation problems have a similar setup. When dealing with reservation of summer cabins or rooms in a hotel, the stations correspond to units of time (days for instance) and seats correspond to the rooms or cabins. For hotel rooms, the room number is often not given to the guest and hence the decision need not be made in real time. For summer cabins the guest may request a specific cabin – if available. In both problems the item of reservation needs not be unique. Furthermore, for both problems, extra issues may be important, for example joint reservation of more than one room that should then be adjacent.

The First Interval strategy for assigning requests to seats is as follows: Assign the request to the first seat that is available for the whole journey. Where first refers to the one with the smallest seat number. If no such seat exists then reject the request.

For an illustration, consider a train with three seats and 14 stations, including the first and the last, and consider the request sequence given in the following table.

Request number	Start	End
1	1	3
2	7	9
3	2	5
4	9	13
5	2	6
6	11	14
7	10	13
8	8	9
9	7	11

The assignment of reservations to seats using the First Interval strategy is shown below. Note that the last request is rejected.

		Inter station intervals												
		1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14
Seat	1	1	1					2	2	4	4	4	4	
	2		3	3	3				8			6	6	6
	3		5	5	5	5					7	7	7	

The Best Interval strategy is as follows: Assign the request to a seat where it leaves as little total free space as possible immediately before and after the request. Ties are broken arbitrarily. If no such seat exists then reject the request.

The assignment of reservations to seats using the Best Interval strategy is shown below. Note that using this strategy, request 8 is placed at seat 2 and thereby the last request can be accepted.

		Inter station intervals												
		1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14
Seat	1	1	1					2	2	4	4	4	4	
	2		3	3	3			9	9	9	9	6	6	6
	3		5	5	5	5			8		7	7	7	

Bibliography

Boyar and Larsen. (1999). The Seat Reservation Problem. *Algorithmica , 25*, 403-417.

Boyar, Krarup, and Nielsen. (2004). Seat Reservation Allowing Seat Changes. *Journal of Algorithms , 52*, 169-192.