Vehicle Routing Problem

Background Information on VBA Programming in Business Economics by Sanne Wøhlk

The Vehicle Routing Problem (VRP) is a distribution problem where a set of customers request a number of goods that are to be delivered from a central depot using a number of vehicles. The problem was first suggested by Dantzig and Ramser, 1959. For further reading about the problem the reader is referred to the classical literature Bodin, Golden, Assad, and Ball, 1983, Bodin, 1991, and Laporte, 1992.

The VRP is formally stated as follows: Given an undirected complete graph $G(N, E, C, Q)$ where $c_{ij} \geq 0$ is the cost of traversing the edge (i, j) and $q_i \geq 0$ is the demand of node i. Let node 1 be a special node called the Depot from which a number of vehicles each of capacity W start at time zero. Find tours such that 1) All nodes with $q_i \geq 0$ are serviced, 2) Vehicle capacities are respected, and 3) The total cost is minimized.

The VRP occurs in many practical applications such as distribution of goods to retailers, collection of milk from farmers to the dairy, or delivery of heating oil to households. Often additional constraints are imposed to the problem. Examples are restrictions on the service time, order of the visits to customers, conflicts of some of the goods to be collected, etc.

Many heuristics and exact algorithms have been proposed for the problem. Here we shortly consider two heuristics:

The idea in the Savings algorithm by Clarke and Wright, 1964 is first to form as many routes as there are nodes in the graph. Each route should start and end in the depot and service exactly one node. Next, the savings that can be obtained by merging two routes are considered. The merging that results in the largest saving is performed if the resulting route is feasible with respect to the constraints of the problem. This process is repeated until no more mergers are possible.

To use the Sweep algorithm by Gillett and Miller, 1974, the nodes of the problem must be given as coordinates in a two dimensional plane. A *base line* is defined with one end in the depot - for instance a vertical line pointing upwards from the depot. For each node, the angle between the base line and the straight line connecting the node to the depot is calculated. The nodes are visited in increasing order with respect to the size of this angle. When the vehicle is full, the corresponding route returns to the depot and a new route is initiated with the next node in the order being the first one to be visited. This corresponds to sweeping the nodes by rotating the base line. Note that the definition of the base line determines the first node to be visited, which is referred to as the *seed*. Clearly the choice of seed influences the final solution significantly.

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