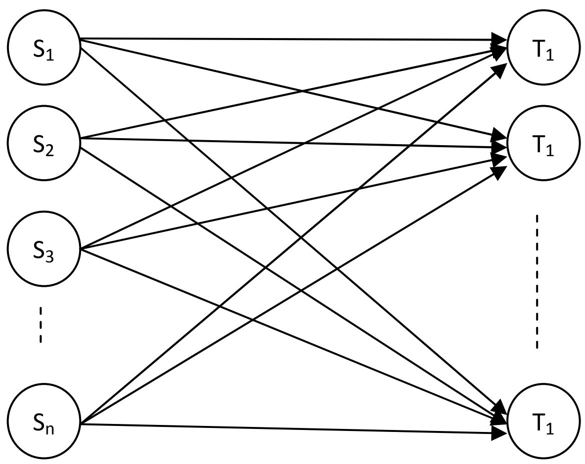
Transportation Problem

Background Information on VBA Programming in Business Economics by Sanne Wøhlk

The transportation problem is the problem of determining the cheapest possible way of transporting a number of goods from a set of suppliers to a set of consumers. Each supplier supplies a certain amount of the goods and each consumer demands a certain amount of goods. In order to determine an optimal transportation plan the amount to be shipped from each supplier to each customer must be determined. Such a transportation network is illustrated below.



For a general definition, let $\mathbb{S}$ be the set of suppliers and let S_i be the amount supplied by supplier $i \in \mathbb{S}$. Similarly, let $\mathbb{T}$ be the set of customers, let T_j be the amount demanded by customer $j \in \mathbb{T}$. Let c_{ij} be the cost of shipping one unit of the goods from supplier *i* to customer *j*. We define the variable x_{ij} to be the amount of goods to be shipped from supplier *i* to customer *j*.

$$\begin{aligned} min \quad & \sum_{i \in \mathbb{S}} \sum_{j \in \mathbb{T}} c_{ij}\, x_{ij} & & \\ s.t. \quad & \sum_{j \in \mathbb{T}} x_{ij} & = S_i \quad & \forall\, i \in \mathbb{S} \\ & \sum_{i \in \mathbb{S}} x_{ij} & = T_j \quad & \forall\, j \in \mathbb{T} \\ & x_{ij} & \geq 0 \quad & \forall\, i \in \mathbb{S}, \forall\, j \in \mathbb{T} \end{aligned}$$

Note that due to the equality constraints, the model requires that $\sum_{i \in \mathbb{S}} S_i = \sum_{j \in \mathbb{T}} T_j$. If there is an excess of supply, a dummy customer can be used such that transportation to this customer is of zero cost.

For illustration purposes, consider a transportation problem with four warehouses demanding 700, 1000, 850, and 500 units respectively. The goods are supplied by three factories supplying 1000, 1500, and 1000 units respectively. Transportation costs are given by the table below.

Factory\Warehouse	1	2	3	4
1	5	3	7	3
2	4	6	3	8
3	3	4	5	4

In the problem there is an excess supply of 450 units which are assigned to a dummy warehouse, d. The mathematical model for the problem is given below. Note how the cost of transporting goods to the dummy warehouse is zero in the model.

$$
\begin{array}{ll}
min & 5x_{11} + 3x_{12} + 7x_{13} + 3x_{14} + \quad 4x_{21} + 6x_{22} + 3x_{23} + 8x_{24} + \quad 3x_{31} + 4x_{32} + 5x_{33} + 4x_{34} \\
s.t. & x_{11} + x_{12} + x_{13} + x_{14} + x_{1d} = 1000 \\
 & +x_{21} + x_{22} + x_{23} + x_{24} + x_{2d} = 1500 \\
 & + x_{31} + x_{32} + x_{33} + x_{34} + x_{3d} = 1000 \\
 & x_{11} + x_{21} + x_{31} = 700 \\
 & x_{12} + x_{22} + x_{32} = 1000 \\
 & x_{13} + x_{23} + x_{33} = 850 \\
 & x_{14} + x_{24} + x_{34} = 500 \\
 & x_{1d} + x_{2d} + x_{3d} = 450 \\
 & x_{ij} \geq 0
\end{array}
$$

The transportation problem is thoroughly discussed in Hillier and Lieberman, 2005. The problem is also treated in Balakrishnan, Render, and Stair, 2007 and in Moore and Weatherford, 2001. The assignment problem can be considered as a variation of the transportation problem and is also discussed in the above references.

Bibliography

Balakrishnan, Render, and Stair. (2007). *Managerial Decision Modeling with Spreadsheets* (2nd ed.). Pearson, Prentice Hall.

Hillier and Lieberman. (2005). *Introduction to Operations Research* (8th Edition ed.). McGraw-Hill.

Moore and Weatherford. (2001). *Decision Modeling with Microsoft Excel* (6th Edition ed.). Prentice Hall.